Fall 2008 STAT 7010

Practice problems

1. An airport limousine can accommodate up to four passengers on any one trip. The company will accept a maximum of six reservations for a trip, and a passenger must have a reservation. From previous records, 20% of all those making reservations do not appear for the trip. Assuming independence wherever appropriate, answer the following questions:
   a) If six reservations are made, what is the probability that at least one individual with a reservation cannot be accommodated on the trip?
   b) If six reservations are made, what is the expected number of available seats when the limousine departs?
   c) Suppose the probability distribution of the number of reservations made is given in the accompanying table:

<table>
<thead>
<tr>
<th>Number of reservations</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>.1</td>
<td>.2</td>
<td>.3</td>
<td>.4</td>
</tr>
</tbody>
</table>

   Let X denote the number of passengers on a randomly selected trip. Obtain the probability distribution of X.

2. The number of requests for assistance received by a towing service follows a Poisson distribution with rate \( \lambda = 4 \) per hour.
   a) Compute the probability that exactly ten requests are received during a particular 2-hour period.
   b) If the operators of the towing service take a 30-min break for lunch, what is the probability that they do not miss any calls for assistance?
   c) How many calls would you expect during their break?

3. Let X be the distribution of time (in sec) headway between two consecutive cars in traffic flow. Suppose the pdf is given by:

   \[
   f(x) = \begin{cases} 
   k x^{-1}, & x > 1 \\
   0, & x \leq 1 
   \end{cases}
   \]

   a) Determine the value of \( k \) for which \( f(x) \) is a legitimate pdf.
   b) Obtain the cumulative distribution function.
   c) Use the cdf from (b) to determine the probability that headway exceeds 2 sec and also the probability that headway is between 2 and 3 sec.
   d) Obtain the mean and standard deviation of the headway time.
   e) What is the probability that headway is within 1 standard deviation of the mean value?
4. The distribution of resistance for resistors of a certain type is known to be normal, with 10% of all resistors having a resistance exceeding 10.256 ohms and 5% having a resistance smaller than 9.671 ohms. What are the mean value and standard deviation of the resistance distribution?

5. The reaction time (in seconds) to a certain stimulus is a continuous random variable with pdf

\[
f(x) = \begin{cases} \frac{3}{2} x^2, & 1 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}
\]

a) What is the probability that reaction time is at most 2.5 sec? Between 1.5 and 2.5 sec?

b) Compute the expected reaction time.

c) Compute the standard deviation of the reaction time.

d) If an individual takes more than 1.5 sec to react, a light comes on and stays on either until one further second has elapsed or until the person reacts (whichever happens first). Determine the expected amount of time that the light remains lit.

6. Let X be the number of packages being mailed by a randomly selected customer at a certain shipping facility. Suppose the distribution of X is as follows:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>.4</td>
<td>.3</td>
<td>.2</td>
<td>.1</td>
</tr>
</tbody>
</table>

a) Consider a random sample of size n=2, (two customers) and let \( \bar{X} \) be the sample mean number of packages shipped. Obtain the probability distribution of \( \bar{X} \).

b) Calculate \( P(\bar{X} \leq 2.5) \).

c) Explain and carry out a simulation to estimate the probability above.

d) Compute the expected value and standard deviation of \( \bar{X} \) in two ways: first using the distribution of \( \bar{X} \), and second using the properties of expected values and variances.

7. One percent of all individuals in a certain population are carriers of a particular disease. A diagnostic test for this disease has 90% detection rate for carriers and a 5% detection rate for noncarriers. Suppose the test is applied independently to two different blood samples from the same randomly selected individual.

a) What is the probability that both tests yield the same result?

b) If both tests are positive, what is the probability that the selected individual is a carrier?

8. A system consists of two components. The probability that the second component functions in a satisfactory manner during its design life is 0.9. The probability that at least one of the two components does so is 0.96, and the probability that both components do so is 0.75. Given that the first component functions in a satisfactory
manner throughout its design life, what is the probability that the second one does also?